Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Wednesday 7 June 2023

Morning (Time: 2 hours)

Paper reference **4MA1/2H**

Mathematics A
PAPER 2H
Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
- Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P72828A

N:1/1/1/1/

P 7 2 8 2 8 A 0 1 2 8

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

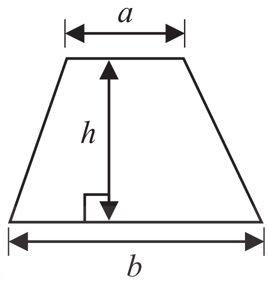
Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

The quadratic equation

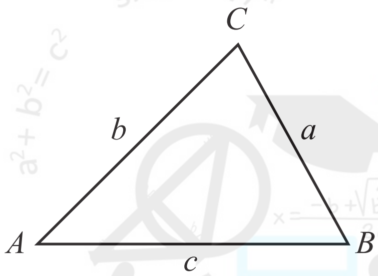
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle ABC

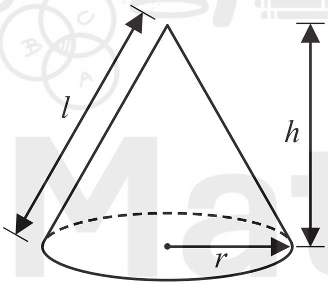
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

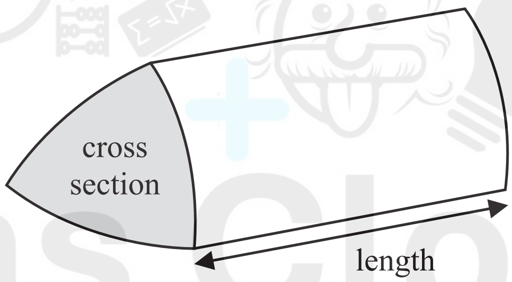
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

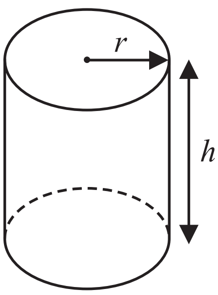
Curved surface area of cone $= \pi rl$



Volume of prism
= area of cross section × length

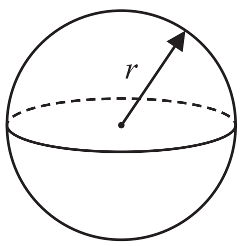


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi rh$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



P 7 2 8 2 8 A 0 2 2 8

Answer ALL TWENTY FIVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Show that $4\frac{2}{3} \div 1\frac{1}{5} = 3\frac{8}{9}$

1) for $4\frac{2}{3}$, we can view $1 = \frac{3}{3}$, so $4 = \frac{3}{3} \times 4 = \frac{12}{3}$.

$\frac{12}{3} + \frac{2}{3} = \frac{14}{3}$

for $1\frac{1}{5}$, we can view $1 = \frac{5}{5}$.

$\frac{5}{5} + \frac{1}{5} = \frac{6}{5}$

2) To add both fractions, we require them to have the same denominator. Lowest common multiple of 3 and 5 = 15.

$\frac{14}{3} \times 5 = \frac{70}{15}$ as $5 \times 3 = 15$

$\frac{6}{5} \times 3 = \frac{18}{15}$ as $3 \times 5 = 15$

3) $\frac{70}{15} \div \frac{18}{15} = \frac{35}{9}$ } Note: we can use the keep, flip, change method for dividing fractions. →

KEEP the $\frac{70}{15}$

FLIP the $\frac{18}{15}$ to $\frac{15}{18}$

CHANGE the $\div$ to a $\times$

Equation becomes: $\frac{70}{15} \times \frac{15}{18} = \frac{1050}{270} = \frac{35}{9}$

4) for $\frac{35}{9}$, we can view $\frac{9}{9} = 1$

$9 \times 3 = 27$

(Total for Question 1 is 3 marks)

so 9 goes into 35 3 times, with 8 remaining $(35 - 27 = 8)$

So expressed as a mixed fraction: $3\frac{8}{9}$

or, once $\frac{14}{3}$ and $\frac{6}{5}$ have been obtained

$\frac{14}{3} \times \frac{6}{5} = \frac{35}{9} = 3\frac{8}{9}$ (simplify using method outlined in step 4.)

or

$\frac{14}{3} \div \frac{6}{5} = \frac{70}{15} \div \frac{18}{15} = \frac{70}{18} = \frac{35}{9} = 3\frac{8}{9}$

(obtain following step 3; ÷ 2 to simplify; obtained following the method in step 4.)

Therefore, $4\frac{2}{3} \div 1\frac{1}{5} = 3\frac{8}{9}$

2 A biased spinner can land on green or on yellow or on brown or on pink.

The table gives the probabilities that, when the spinner is spun, it will land on green or on yellow or on brown.

Colour	**green**	**yellow**	**brown**	**pink**
Probability	0.32	0.13	0.28	0.27

Timucin spins the spinner 200 times.

Work out an estimate for the number of times the spinner lands on pink.

To find the probability of the spinner landing on pink, do 1 minus the known probabilities.

$1 - (0.32 + 0.13 + 0.28)$

$1 - 0.73 = 0.27$

The spinner is spun 200 times. To estimate the number of times it lands on pink, multiply the probability by 200.

$200 \times 0.27 = 54$

Or, calculate the spinner estimates for the other colours and subtract the values from 200.

$200 \times 0.32 = 64$

$200 \times 0.13 = 26$

$200 \times 0.28 = 56$

$200 - (64 + 26 + 56)$

$= 200 - 146 = 54$

54

(Total for Question 2 is 3 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

3 *ABCD* is a trapezium.

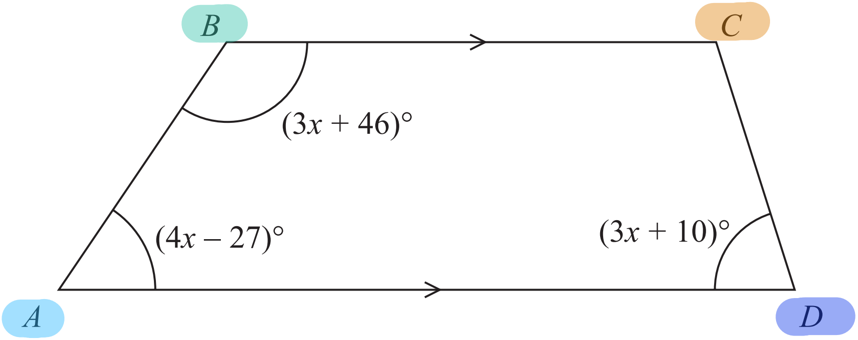


Diagram **NOT** accurately drawn

BC is parallel to *AD*

Find the size of the largest angle inside the trapezium.

Trapezium angle rules tells us that angle A and angle B will add up to 180°.

Therefore, A + B = 180

$(4x-27)+(3x+46) = 7x+19$

$7x + 19 = 180$

$7x = 180 - 19$

$7x = 161$

$x = \frac{161}{7}$

$x = 23$

plug 23 back into the angle equations.

A) $4x - 27 = 4(23) - 27 = 92 - 27 = 65°$

B) $3x + 46 = 3(23) + 46 = 69 + 46 = 115°$

D) $3x + 10 = 3(23) + 10 = 69 + 10 = 79°$

we know the angles in a trapezium must sum to 360°, so we can find angle C by subtracting A, B and D from 360.

$360 - (65 + 115 + 79)$

$360 - 259 = 101°$

From the calculated angles, we can see that angle B (115°) is the largest.

.......................115 °

(Total for Question 3 is 4 marks)

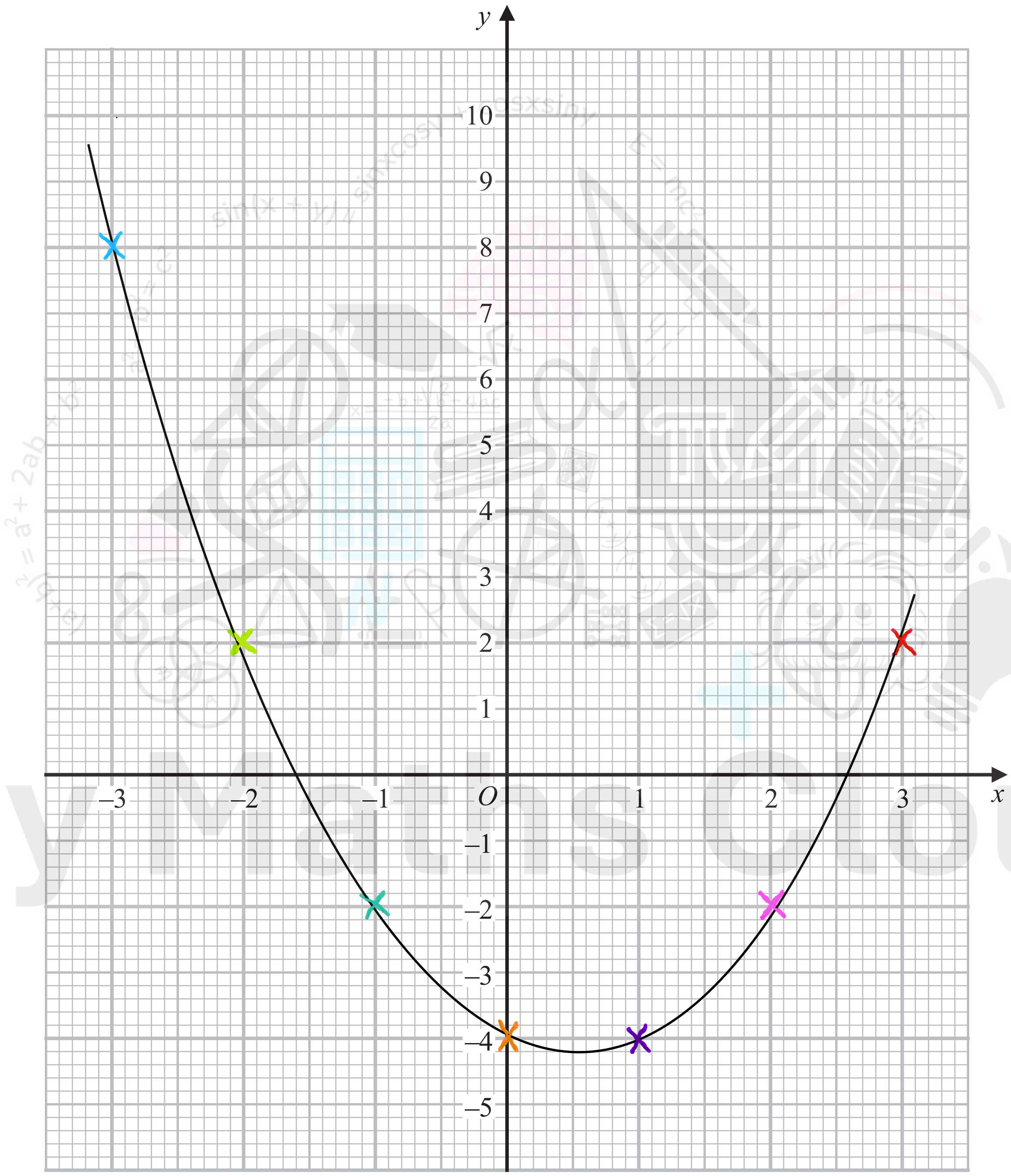
4 (a) Complete the table of values for $y = x^2 - x - 4$

$(-3)^2 - (-3) - 4 = 8$
$(-1)^2 - (-1) - 4 = -2$
$0^2 - 0 - 4 = -4$
$2^2 - 2 - 4 = -2$
$3^2 - 3 - 4 = 2$

x	–3	–2	–1	0	1	2	3
y	8	2	-2	-4	–4	-2	2

(2)

(b) On the grid below, draw the graph of $y = x^2 - x - 4$ for values of x from –3 to 3



(2)

(Total for Question 4 is 4 marks)

P 7 2 8 2 8 A 0 6 2 8

5 Nancy has some coins with a total value of 85 pence.
She has only 2 pence coins and 5 pence coins.
The ratio

number of 2 pence coins : number of 5 pence coins = 1 : 3

Nancy has more 5 pence coins than 2 pence coins.

How many more?

let x = number of coins.

$2 \times 1 = 2$

$5 \times 3 = 15$

$2x + 15x = 85$

$17x = 85$

$x = \frac{85}{17}$

$x = 5$

$10 \div 2 = 5$ 2p coins.

$75 \div 15 = 15$ 5p coins

$2x : 15x$

$2 \times 5 = 10$

$15 \times 5 = 75$

So the ratio becomes:

2p coins : 5p coins

10 : 75

Now ÷ by the value of the coins (2p and 5p)

$10 \div 2 = 5$ 2p coins

$75 \div 5 = 15$ 5p coins

Therefore, the difference is: $15 - 5 = 10$ coins

So Nancy has 10 more 5p coins.

.......................10

(Total for Question 5 is 4 marks)

6 (a) Write 76 000 000 in standard form.

value must be less than 10.

76 000 000

$= 7.6 \times 10^7$ ← moved 7 places

Note: The 7 is positive as 76,000,000 is not a decimal value.

.......................7.6×10^7

(1)

(b) Write 5.4×10^{-4} as an ordinary number.

$5.4 = 0.00054$

Note: The −4 is negative, so we move the decimal point to the left.

.......................0.00054

(1)

(Total for Question 6 is 2 marks)

P 7 2 8 2 8 A 0 7 2 8

7

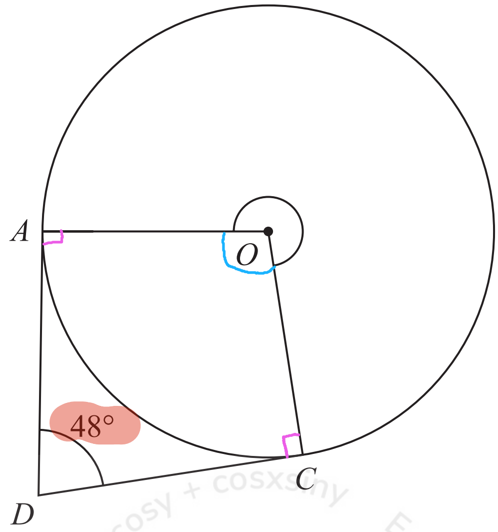


Diagram **NOT** accurately drawn

A and *C* are points on a circle, centre *O*
DA is the tangent to the circle at *A* and *DC* is the tangent to the circle at *C*

Angle *ADC* = 48°

Work out the size of reflex angle *AOC*

Firstly recognise that $D\hat{A}O$ and $D\hat{C}O$ are right angles (90°).
Sum of the internal angles of DAOC = 360°.
Internal angle $A\hat{O}C = 360 - 90 - 90 - 48 = 132°$
Size of the reflex angle = 360 − 132 = 228°

228 °

(Total for Question 7 is 3 marks)

8 Charlotte buys a painting for \$680
The value of the painting increases by 4% each year.

Work out the value of the painting at the end of 3 years.
Give your answer correct to the nearest \$

As the value increases by 4% each year, we can represent this as the multiplier: 1.04 $(1 + 0.04 = 1.04)$

As the time duration is 3 years, the multiplier is raised to the power of 3.

So the multiplier becomes: 1.04^3

So to calculate the value of the painting: $680 \times 1.04^3 = 764.91$

Rounded to the nearest \$ = \$765

\$ 765

(Total for Question 8 is 3 marks)

---

9 Change a speed of 27 kilometres per hour to a speed in metres per second.

Think about the question as 2 conversions. First converting the distance units, and then the time units.

Converting the distance:

There are 1000 meters in 1 kilometer.

$27 \times 1000 = 27{,}000$

Converting the time:

There are 60 minutes in a hour and 60 seconds in a minute. So divide by 60×60 to convert from meters per hour to meters per second.

$= \frac{27000}{60 \times 60} = 7.5$

7.5 m/s

(Total for Question 9 is 3 marks)

10 Team **A** and Team **B** take part in a quiz league.

After 11 rounds, Team **A** has a mean score per round of 17
After 9 rounds, Team **B** has a mean score per round of 18

Both teams take part in a further round.
After this round, both teams have a mean score per round of 18.5

In the further round, Team **A** scored more points than Team **B**.

How many more?

Step 1: Calculate the current points for both teams.
Team A: 11 × 17 = 187 points
Team B: 9 × 18 = 162 points

Step 2: Calculate the points after the further round.
Team A: 12 × 18.5 = 222 points
Team B: 10 × 18.5 = 185 points

Note: 12 and 10 are used as these are the total rounds with the further round included (11+1=12) (9+1=10).

Step 3: Subtract the points after Step 1 from the points after Step 2.
Team A: 222 − 187 = 35
Team B: 185 − 162 = 23

Step 4: Subtract Team B's points from Team A's points from Step 4.
35 − 23 = 12 more points

12

(Total for Question 10 is 4 marks)

11 Here is a 9-sided regular polygon *ABCDEFGHJ*, with centre *O*

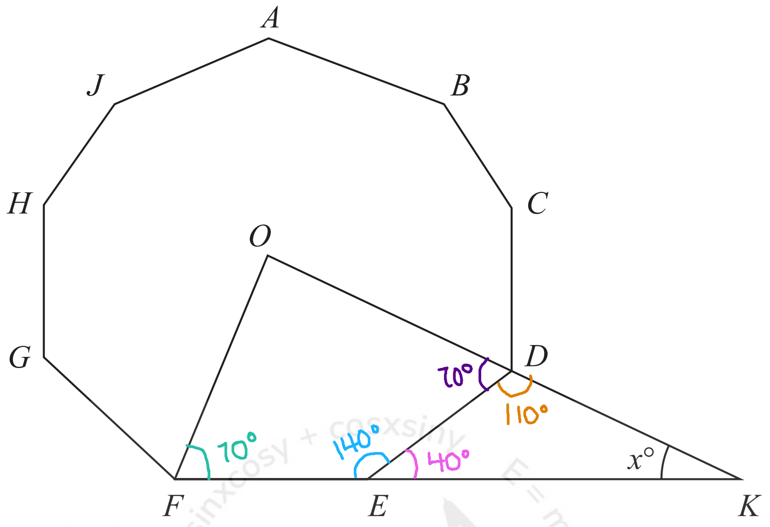


Diagram **NOT** accurately drawn

ODK and *FEK* are straight lines.

Work out the value of *x*

DÊK is an external angle of the polygon.

External angle formula : 360° / number of sides

DÊK = 360° / 9

DÊK = 40°

Angles on a straight line = 180°, so FÊD = 180 − 40

FÊD = 140°

If the internal angles = 140°, then OF̂K = 140 ÷ 2

OF̂K = 70°

Now consider the opposite angles, so OF̂K = OD̂E

so OD̂E = 70°

Using angles on a straight line = 180°

KD̂E = 180 − 70

KD̂E = 110°

Angles in a triangle = 180°

so $x = 180 - 110 - 40$

$x = 30°$

$x =$ 30°

(Total for Question 11 is 3 marks)

12 The diagram shows right-angled triangle *ABD*

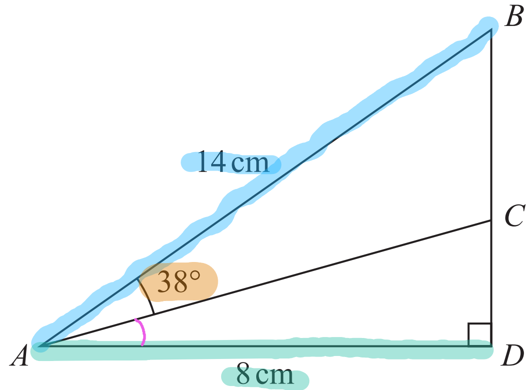


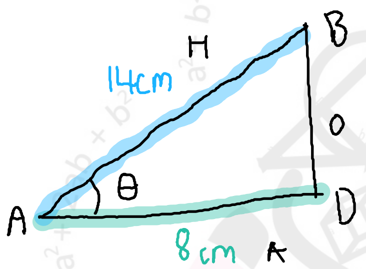
Diagram **NOT** accurately drawn

$AB = 14$ cm $\quad AD = 8$ cm

C is the point on *BD* such that angle $BAC = 38°$

Work out the length of *CD*
Give your answer correct to 3 significant figures.

(1)



$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{8}{14}$

$\theta = \cos^{-1}\left(\frac{8}{14}\right)$

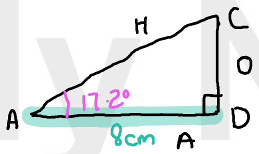
$\theta = 55.15...°$

(2)

$C\hat{A}D = 55.15... - 38$

$C\hat{A}D = 17.2°$

(3)



$\tan(17.2) = \frac{O}{8}$

$O = \tan(17.2) \times 8$

$O = 2.476...$

$\therefore O = CD$

$\therefore CD = 2.476...$

rounded to 3 significant figures $= 2.47$ cm

2.47 cm

(Total for Question 12 is 4 marks)

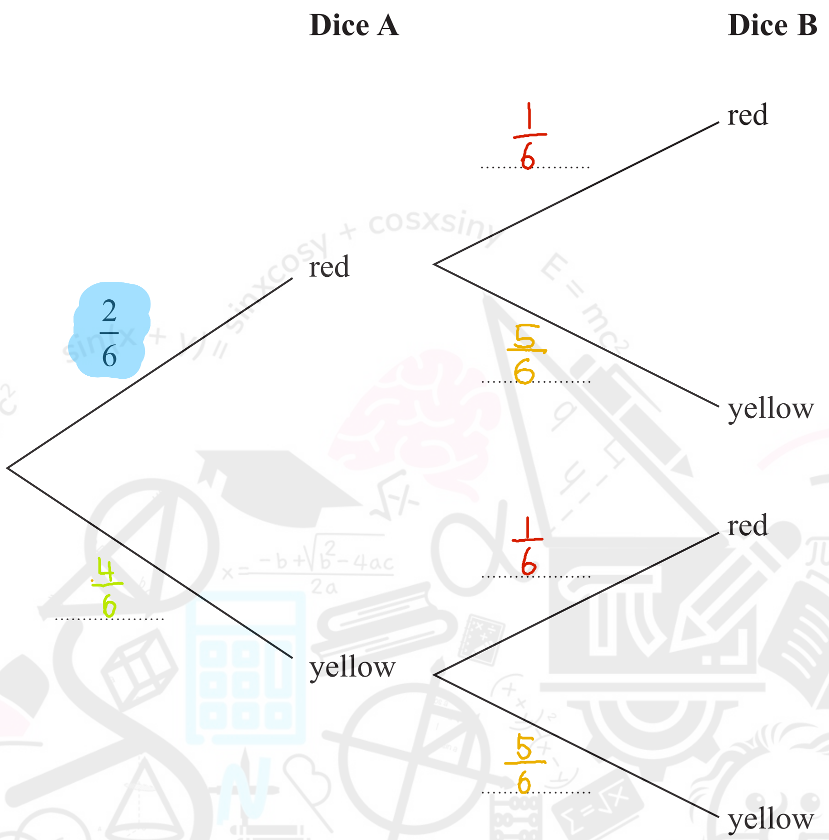
P 7 2 8 2 8 A 0 1 2 2 8

13 Narin has two fair 6-sided dice.

Dice **A** has 2 red faces and 4 yellow faces.
Dice **B** has 1 red face and 5 yellow faces.

Narin is going to throw each dice once.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both dice land on yellow.

Both branches must = 1. We can express this as $\frac{6}{6}$,
So the probability of Dice A landing on a yellow face = $\frac{6}{6} - \frac{2}{6} = \frac{4}{6}$ (This can also be expressed as $\frac{2}{3}$)
Dice B has 1 red face and 5 yellow faces.
So the probability it lands on red = $\frac{1}{6}$
Probability it lands on yellow = $\frac{5}{6}$

Probability both land on yellow: $\frac{4}{6} \times \frac{5}{6} = \frac{5}{9}$

$\frac{5}{9}$

(2)

(Total for Question 13 is 4 marks)

14

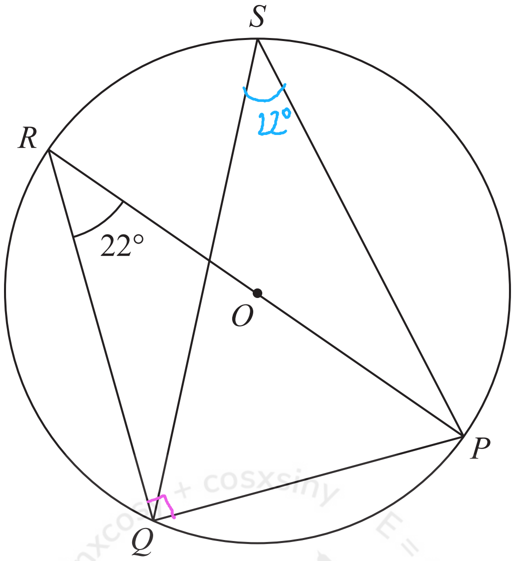


Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle, centre O
ROP is a diameter of the circle.
Angle $PRQ = 22°$

(a) (i) Find the size of angle RQP

90 °
(1)

(ii) Give a reason for your answer.

Angle in semicircle is 90° or triangle in semicircle is 90° or angle at the centre is double the angle at the circumference or angle at the circumference is half the angle at the centre.
(1)

(b) (i) Find the size of angle PSQ

22 °
(1)

(ii) Give a reason for your answer.

Angles in the same segment are equal or angles at the circumference subtended from the same arc of the circle or angles are on the same chord.
(1)

(Total for Question 14 is 4 marks)

P 7 2 8 2 8 A 0 1 4 2 8

$15 \times \frac{3}{5} = 9$

15 (a) Simplify fully $(32a^{15})^{\frac{3}{5}}$

Think of it in parts:

$32^{\frac{3}{5}} = 8$

$a^{15 \times \frac{3}{5}} = a^9$

Combining them:

$= 8a^9$

$8a^9$

(2)

(b) Express $\left(\frac{1}{10x}\right)^{-3}$ in the form px^n where p and n are integers.

As we have a -3, then we can use the rule: $a^{-b} = \frac{1}{a^b}$

So $\left(\frac{1}{10x}\right)^{-3}$ becomes $\frac{1}{\left(\frac{1}{10x}\right)^3}$

$10x^3 = 1000x^3$

So the fraction becomes: $\frac{1}{\left(\frac{1}{1000x^3}\right)} = 1000x^3$

Or, recognise that -3 can 'flip' the fraction + become positive.

$\left(\frac{1}{10x}\right)^{-3} = \left(\frac{10x}{1}\right)^3 = (10x)^3$

$1000x^3$

(2)

(c) Solve $\frac{1-2y}{3} = \frac{4}{5} - \frac{2y-1}{2}$

Show clear algebraic working.

The lowest common multiple between 3, 5 and 2 is 30.
Multiply each fraction to get 30 as the denominator.

$\frac{1-2y}{3} \times 10 = \frac{10-20y}{30}$ | $\frac{4}{5} \times 6 = \frac{24}{30}$ | $\frac{2y-1}{2} \times 15 = \frac{30y-15}{30}$

As each fraction has a common denominator, we can cancel it.

$\frac{10-20y}{30} = \frac{24}{30} - \frac{30y-15}{30}$ becomes: $10-20y = 24 - 30y - 15$

(+30y both sides) $10 + 10y = 24 + 15$

Note: This becomes a plus as $24 - (-15) = 24 + 15$

$10 + 10y = 39$

(−10 both sides) $10y = 29$

(÷10 both sides) $y = 2.9$

$y =$ 2.9

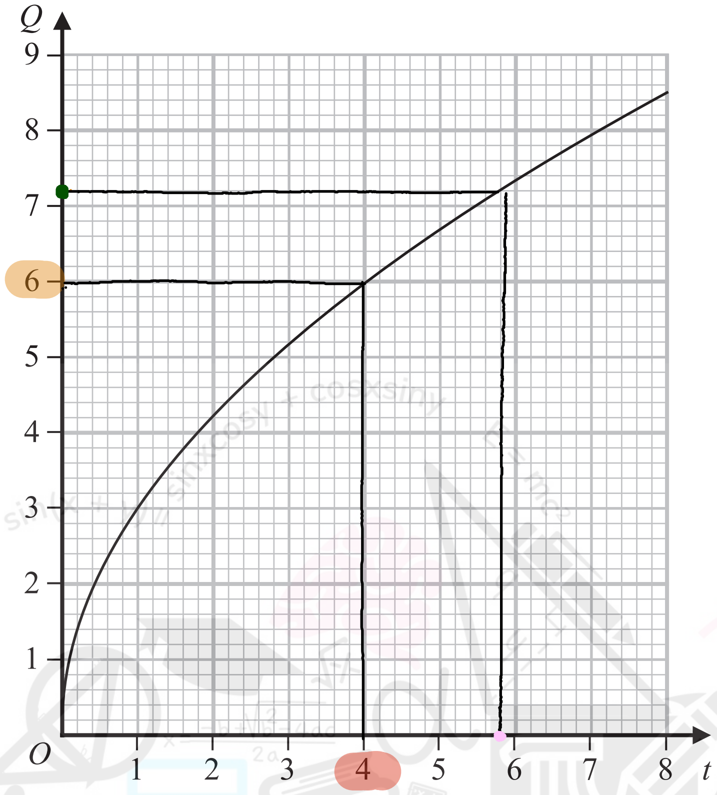
(3)

(Total for Question 15 is 7 marks)

DO NOT WRITE IN THIS AREA

16 Q is directly proportional to $\sqrt{t}$

The graph shows the relationship between Q and t for $0 < t < 8$



(a) Find a formula for Q in terms of t

Note: As Q is directly proportional to √t, the equation becomes k × √t.

Q = k√t or Q ∝ k√t

Find the point on the graph where Q and t are nicer, whole values.

when Q = 6 and t = 4

Plug the values into Q = k√t

6 = k√4

√4 = 2, so the equation becomes:

6 = 2k, (÷ 2)

3 = k

Q = 3√t

(3)

Q is increased by 20%

(b) Find the percentage increase in t

when Q = 6, increasing by 20% ∴ 6 × 1.2 = 7.2

finding Q = 7.2 on the graph and drawing to the point where it meets the line gives t = 5.8

Percentage increase formula: (new value − old value / old value) × 100

= (5.8 − 4 / 4) × 100 = 45

45 %

(2)

Note: Answers in the range of 43-45% will be accepted.

(Total for Question 16 is 5 marks)

17 (a) Expand and simplify $(x + 6)(3x - 2)(x + 6)$

Start by expanding $(x+6)$ and $(3x-2)$ using the box method:

	x	$+6$
$3x$	$3x^2$	$18x$
-2	$-2x$	-12

$= (3x^2 + 16x - 12)(x+6)$

collecting like terms of $12x$ and $-2x$.

Apply the same method to expand $(3x^2+16x-12)$ and $(x+6)$.

	$3x^2$	$+16x$	-12
x	$3x^3$	$+16x^2$	$-12x$
$+6$	$18x^2$	$96x$	-72

$= 3x^3 + 34x^2 + 84x - 72$

collecting like terms of $16x^2$ and $18x^2$

collecting like terms of $96x$ and $-12x$

$3x^3 + 34x^2 + 84x - 72$

(3)

(b) Make e the subject of $w = \sqrt{\dfrac{e+g}{ef-d}}$

To remove the square root from both sides, square both sides.

$w^2 = \dfrac{e+g}{ef-d}$

multiply by $ef-d$

$w^2(ef-d) = e+g$

Expand the brackets

$w^2ef - w^2d = e + g$

minus e (we want everything with an e term on the same side)

$w^2ef - w^2d - e = g$

Add w^2d to both sides (we want everything without an e term on the same side)

$w^2ef - e = g + w^2d$

Factorise e out on the left hand side

$e(w^2f - 1) = g + w^2d$

Note: 1 is in the bracket as $e = 1e$.

Divide by (w^2f-1)

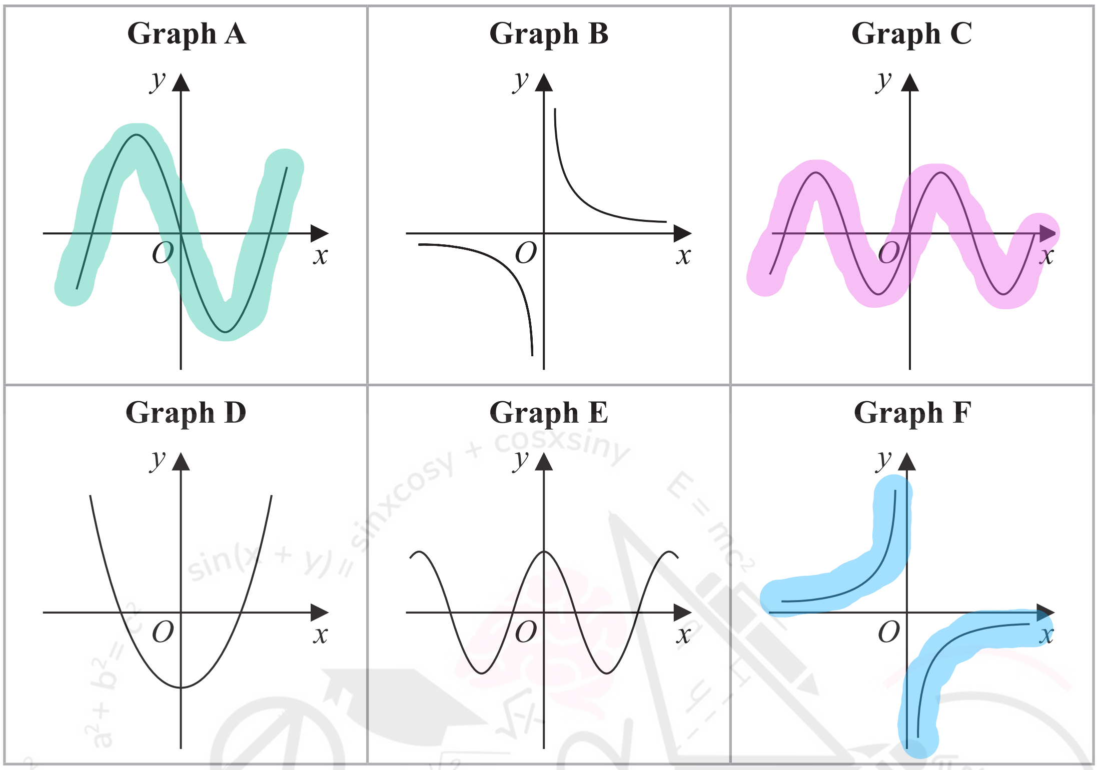
$e = \dfrac{g+w^2d}{w^2f-1}$

$e = \dfrac{g+w^2d}{w^2f-1}$

(4)

(Total for Question 17 is 7 marks)

18 Here are 6 graphs.



Complete the table below with the letter of the graph that could represent each given equation.

Write your answers on the dotted lines.

Equation	Graph
$y = \sin x$	C
$y = -\dfrac{3}{x}$	F
$y = 4x^3 - 5x$	A

notice destinctive sinx shape of graph C.

Any interger ÷ x produces "⌐L" shaped graph, there is a − infront of the $-\frac{3}{x}$, so the shape is reflected, producing a "⌐r" shaped graph.

Cubic graphs produce the "~" shape. Notice that if you plug in x = 0, we get: $4(0)^3 - 5(0) = 0$, therefore when x=0, y=0, so the graph passes through the origin.

(Total for Question 18 is 3 marks)

19 Express $3x^2 - 6x + 5$ in the form $a(x - b)^2 + c$

This technique is called completing the square.

Factor out 3.

$3[x^2 - 2x] + 5$

Note: 2 is inside the bracket as factoring out 3 means $6 \div 3 = 2$.

Complete the square on $[x^2 - 2x]$

$3[(x-1)^2 - 1] + 5$

Note: 1 is inside the round brackets as we always ÷ the x term (2) by 2. $2 \div 2 = 1$.

Note: -1 is outside the bracket as we always subtract the square of the integer (1). $1^2 = 1$

multiply by 3

$3 \times 1 = 3$

$3(x-1)^2 - 3 + 5$

collect the like terms of -3 and $+5$

$3(x-1)^2 + 2$

$3(x-1)^2 + 2$

(Total for Question 19 is 3 marks)

20 There are 12 counters in a bag.

3 of the counters are red
9 of the counters are green

Ameya, Jack and Ella each take at random one counter from the bag.

Work out the probability that at least one red counter is still in the bag.

The only way at least 1 red counter is not left in the bag is if all 3 red counters are chosen.

(3 reds chosen): $\frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} = \frac{1}{220}$

As we want the probability that at least 1 red counter is still in the bag, take $\frac{1}{220}$ from 1.

$1 - \frac{1}{220} = \frac{219}{220}$

or, work out each senario and add them up:

(2 red, 1 green) $= \left(\frac{3}{12} \times \frac{2}{11} \times \frac{9}{10}\right) \times 3 = \frac{27}{220}$

(1 red, 2 green) $= \left(\frac{3}{12} \times \frac{9}{11} \times \frac{8}{10}\right) \times 3 = \frac{108}{220}$

(3 green) $= \left(\frac{9}{12} \times \frac{8}{11} \times \frac{7}{10}\right) = \frac{84}{220}$

$\frac{27}{220} + \frac{108}{220} + \frac{84}{220} = \frac{219}{220}$

Note: we multiply the first 2 equations by 3 as there are 3 different ways which the senario can occur.

e.g: A J E
R, R, G or
R, G, R or
G, R, R

$\frac{219}{220}$

(Total for Question 20 is 3 marks)

21 Solve the simultaneous equations

$$2x^2 + 3y^2 = 11$$
$$x = 3y - 1$$

Show clear algebraic working.

Subsicute $x = 3y - 1$ into $2x^2 + 3y = 1$

$2(3y-1)^2 + 3y^2 = 11$

	$3y$	-1
$3y$	$9y^2$	$-3y$
-1	$-3y$	1

$= 9y^2 - 6y + 1$

$2(9y^2 - 6y + 1) + 3y^2 = 11$

Expand the brackets

$18y^2 - 12y + 2 + 3y^2 = 11$

Collect like terms

$21y^2 - 12y + 2 = 11$

-11 from both sides

$21y^2 - 12y - 9 = 0$ (a, b, c)

Using the quadratic equation, where $a = 21$, $b = -12$, $c = -9$.

$y = \frac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2(a)}$

$y = \frac{-(-12) \pm \sqrt{12^2 - 4(21)(-9)}}{2(21)}$

$y = \frac{12 + \sqrt{900}}{42}$ or $y = \frac{12 - \sqrt{900}}{42}$

$y = \frac{3}{7}$ $\qquad$ $y = 1$

Plugging back into $x = 3y - 1$

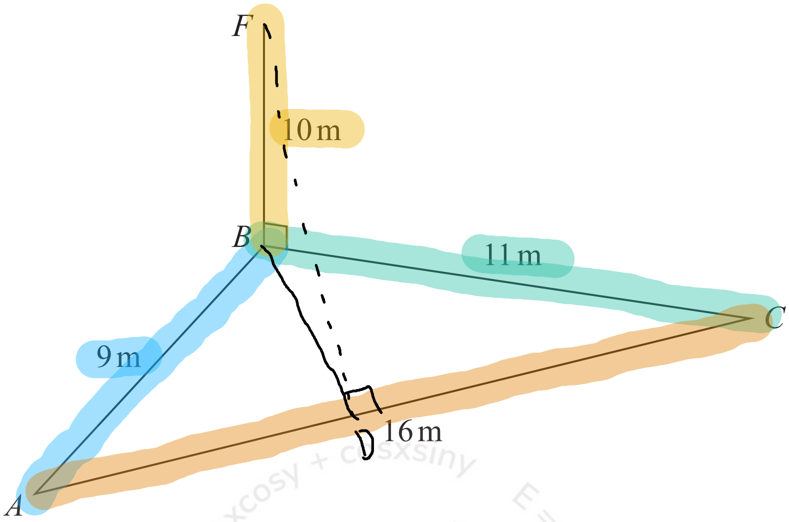
$x = 3(3/7) - 1$ or $x = 3(1) - 1$

$x = \frac{-16}{7}$ $\qquad$ $x = 2$

$y = -3/7$, $x = -16/7$ and $y = 1$, $x = 2$

(Total for Question 21 is 5 marks)

22 The diagram shows a triangle *ABC* and a flagpole *BF*



A, *B* and *C* are points on horizontal ground.

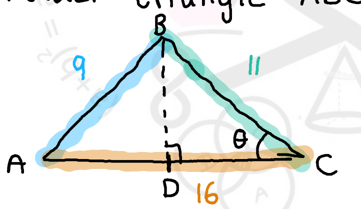
BF is vertical.

AB = 9 m *BC* = 11 m *AC* = 16 m *BF* = 10 m

D is the point on *AC* such that angle *BDC* = 90°

Work out the size of the angle of elevation of the point *F* from the point *D*
Give your answer correct to one decimal place.

Consider triangle ABC:



Using cosine rule to find θ:

$a^2 = b^2 + c^2 - 2bc\cos A$

$9^2 = 11^2 + 16^2 - 2(11)(16)\cos\theta$

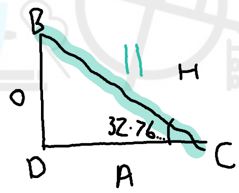
$\cos\theta = \left(\frac{11^2 + 16^2 - 9^2}{2(11)(16)}\right)$ ①

$\theta = \cos^{-1}(①)$

$\theta = 32.763\ldots°$

$\therefore BCA = 32.763\ldots°$

Now consider triangle BCD:



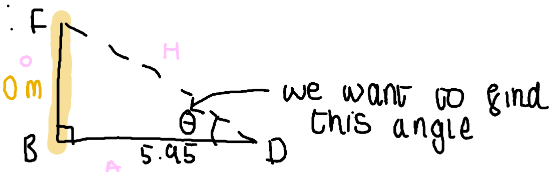
$\sin\theta = \frac{O}{H}$

$\sin 32.76 = \frac{BD}{11}$

$BD = \sin 32.76\ldots \times 11$

$BD = 5.95\text{m}$

Triangle FDB:



$\tan FDB = \frac{O}{A}$

$\tan FDB = \frac{10}{5.95}$

$FDB = \tan^{-1}\left(\frac{10}{5.95}\right)$ $FDB = 59.2°$

59.2 °

(Total for Question 22 is 5 marks)

Turn over for Question 23

23 The diagram shows a cuboid with a square cross section.

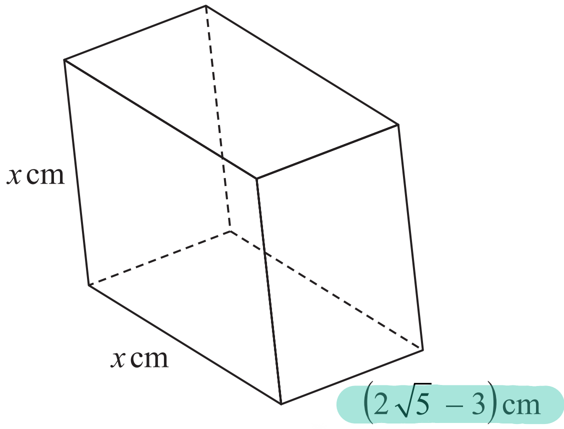


Diagram **NOT** accurately drawn

The volume of the cuboid is $(13 + 6\sqrt{5})$ cm^3

Without using a calculator, find the value of x
Give your answer in the form $a + \sqrt{b}$ where a and b are integers.
Show your working clearly.

$x^2 = \frac{\text{volume}}{\text{side length}}$

$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3}$

Rationalising the denominator

$\frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3} \times \frac{2\sqrt{5} + 3}{2\sqrt{5} + 3}$

	13	$6\sqrt{5}$
$2\sqrt{5}$	$26\sqrt{5}$	60
3	39	$18\sqrt{5}$

	$2\sqrt{5}$	3
$2\sqrt{5}$	20	$6\sqrt{5}$
-3	$-6\sqrt{5}$	-9

$= \frac{26\sqrt{5} + 18\sqrt{5} + 39 + 60}{20 - 9}$

Collecting like terms

$x^2 = \frac{99 + 44\sqrt{5}}{11}$

Simplifying

$x^2 = 9 + 4\sqrt{5}$

$x = 2 + \sqrt{5}$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 7 2 8 2 8 A 0 2 4 2 8

$x =$ $2 + \sqrt{5}$

(Total for Question 23 is 4 marks)

Turn over for Question 24

Drawing a rough sketch can help us visualise the question

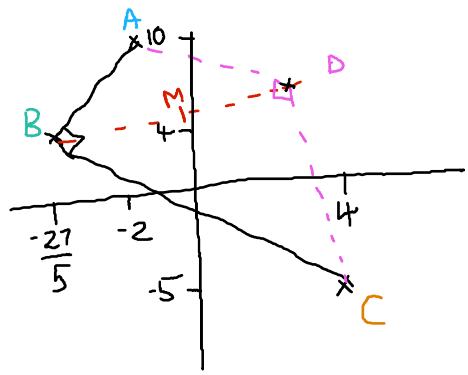
24 *ABCD* is a kite with $AB = AD$ and $CB = CD$

A is the point with coordinates $(-2, 10)$

B is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

C is the point with coordinates $(4, -5)$

Work out the coordinates of *D* (x, y)



First find the midpoint of BD:

Start by finding the gradient of AC:

$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{10 - (-5)}{-2 - 4}$

$m_1 = -\frac{5}{2}$

Plugging m_1 into the equation of a line:

$y = mx + c$

$10 = -\frac{5}{2}(-2) + c$

$c = 10 - \left(-\frac{5}{2}(-2)\right)$

$c = 5$

$\therefore y = -\frac{5}{2}x + 5$ ①

Now find the gradient of BD. As AC and BD are perpendicular to each other, we can use:

$m_1 \times m_2 = -1$

$-\frac{5}{2} \times m_2 = -1$

$m_2 = \frac{2}{5}$

gradient of BD $= \frac{2}{5}$

Plugging m_2 into the equation of a line:

$y = m_2 x + c$

$4 = \frac{2}{5}\left(-\frac{27}{5}\right) + c$

$c = 4 - \left(\frac{2}{5}\left(-\frac{27}{5}\right)\right)$

$c = \frac{154}{25}$

$y = \frac{2}{5}x + \frac{154}{25}$ ②

Making ① = ② and solving for x:

$-\frac{5}{2}x + 5 = \frac{2}{5}x + \frac{154}{25}$

$-\frac{29}{10}x + 5 = \frac{154}{25}$

$-\frac{29}{10}x = \frac{29}{25}$

$x = -\frac{2}{5}$

Plugging x into ①:

$y = -\frac{5}{2}x + 5$

$y = -\frac{5}{2}\left(-\frac{2}{5}\right) + 5$

$y = 6$

midpoint of BD $= \left(-\frac{2}{5}, 6\right)$

Plugging in the coordinates of B and the midpoint into the equation of the midpoint to find D:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

x coordinate of D $= \frac{x_1 + \left(-\frac{27}{5}\right)}{2} = -\frac{2}{5}$

$x_1 = 4.6$

y coordinate of D:

$\frac{y_1 + 4}{2} = 6$

$y_1 = 8$

$\therefore$ coordinates of D: $(4.6, 8)$

(4.6 , 8)

(Total for Question 24 is 6 marks)

Turn over for Question 25

25 A solid sphere has a radius of 2.8 centimetres, correct to 1 decimal place.
The sphere has a mass of $M\pi$ grams, where $M = 260$ correct to 2 significant figures.

Work out the upper bound for the density of the sphere.
Give your answer in g/cm^3 correct to 2 decimal places.
Show your working clearly.

For the upper bound of the density, we need to use the lower bound of the radius and the upper bound of the mass.

Lower bound of radius: 2.75
Upper bound of volume: 265

Volume of sphere: $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(2.75)^3$

$= \frac{1331}{48}\pi$

Density $= \frac{\text{mass}}{\text{volume}}$

Density $= \frac{265\pi}{1331/48\,\pi}$

Density = 9.56 (2 decimal places)

9.56 g/cm^3

(Total for Question 25 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA